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RADIATIVE ENERGY TRANSFER THROUGH NON-GRAY GAS LAYERS OF SMALL OPTICAL THICKNESS*

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Abstract—A theoretical investigation of the energy transfer in a non-gray medium bounded by two flat surfaces is conducted. The theoretical model employs a spectral and temperature dependent absorption coefficient instead of the classical gray gas assumption. The study includes the effects of surface emissivities, and transient conditions. The analysis is carried out under the restriction that the optical depth, τ_ν, is less than one. Expressions for both the temperature distribution and heat flux are presented. The results demonstrate the inadequacies of the gray gas model.

NOMENCLATURE

a_ν, spectral absorption coefficient;
B_ν, Planck's function
$$B_\nu(T) = \frac{2h\nu^3}{c^2[\exp(h\nu/\bar{k}T) - 1]};$$
c_v, thermal heat capacity;
$E_n(x)$, equation (6);
e, blackbody emissive power σT^4;
f, function defined following equation (15);
F, dimensionless function
$$F = \pi^4[g_r - (\sigma T_1^4 - \sigma T_2^4)]/5[\sigma(h\nu/\bar{k})^3\,\beta l T_2];$$
g, function defined following equation (10);
h, Planck's constant;
I_ν, intensity of radiation;
$\bar{k}$, Boltzmann's constant;
k, thermal conductivity;
$k_{p,n}$, Planck mean absorption coefficient
$$k_{p,n} = \frac{\pi\int_0^\infty B_\nu(T_n)\,k_\nu \mathrm{d}\nu}{\sigma T_n^4};$$
$k^2_{m,n}$, mean absorption coefficient [equation (13)];
$k^2_{q,n}(\ln k_{q,n})$, mean absorption coefficient group [equation (14)];
k_ν, spectral portion of absorption coefficient, $a_\nu = \beta(T)\,k_\nu$;
l, distance between surfaces;
q, energy flux;
R, radiosity;
T, absolute temperature;
T_0, first approximation for temperature;
t, time;
y, coordinate normal to flat surfaces.

Greek symbols

γ, Euler–Masheroni constant ($\gamma = 0.5772\ldots$);
β, temperature dependence of absorption coefficient, $a_\nu = \beta(T)\,k_\nu$;
ε, surface emissivity;
μ, direction cosine, $\mu = \cos\theta$;
ρ, density of gas;
τ, opacity, $\tau = \int_0^y a_\nu\,\mathrm{d}y$;
τ_0, total opacity, $\tau_0 = \int_0^l a_\nu\,\mathrm{d}y$.

Subscripts

1, lower surface;
2, upper surface;
ν, frequency;
g, gray medium;
p, Planck mean coefficient;

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m,	mean absorption coefficient;
q,	mean absorption coefficient;
r,	radiation;
c,	conduction.

INTRODUCTION

THIS paper is concerned with the analytical prediction of heat-transfer rates when the radiation field can be considered as one dimensional and the opacity is between zero and one. The analysis considers radiation dominant energy transfer and includes the effects of a spectral and temperature dependent absorption coefficient, surface emissivities, and transient conditions. To date, little analytical work has been done which considers these effects for this range of optical depths. However, a number of numerical calculations have been performed for steady-state situations which include conduction and surface emissivity effects. Summaries of many of these investigations have been compiled by Hottel [1], Viskanta [2] and Cess [3]. Since these reviews were made, numerous publications on the subject of one dimensional radiative energy transfer have appeared in the literature. The papers of Goulard [4], Heaslet and Warming [5] and Grief [12] provide references to most of these.

The purpose of this article is to derive expressions, for the temperature and heat flux, which include an absorption coefficient that varies with both temperature and frequency. The value of such an investigation is that it yields considerable insight into the limitations of a gray gas analysis which cannot be obtained from numerical computations. In addition it answers the question of whether a gray gas analysis can be adapted to a real gas by substituting an appropriate Planck mean absorption coefficient. The classical geometry considered is that of two parallel flat surfaces, infinite in extent, which are separated a distance l (Fig. 1). The surface located at $y = 0$ has a temperature T_1 and that at $y = l$ a temperature T_2. For the purpose of providing bounds we will assume $T_2 > T_1$. The medium between the bounding surfaces is allowed to absorb and emit radiation and the surfaces are permitted to have emissivities different from one. For this investigation it is assumed that the absorbing medium can be described by an absorption coefficient of the form $a_\nu = \beta(T)\, k_\nu$ and that the opacity $\tau_{\nu,0} = \int_0^l a_\nu \, dy$, satisfies the condition $\tau_{\nu,0} < 1$. The real part of the refractive index is assumed to be one throughout the analysis. For one portion of the investigation the plate temperature T_1 is allowed

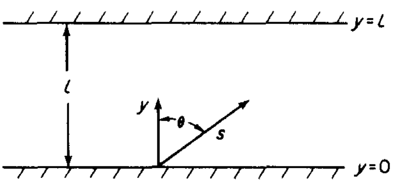

FIG. 1. Diagram showing relationship of radiation transport path, s, to normal coordinate y.

to vary with time. This is an effect which, to the author's knowledge, has not been investigated for problems of this type.*

The analysis proceeds by first writing the equations describing the radiation transport between the two plates. Next, the solution for the temperature in the steady-state radiation only case is developed by a perturbation procedure. In the course of the development new mean absorption coefficients arise which demonstrate that employing a simple Planck mean is inadequate for describing the temperature distribution. The temperature distribution is then employed to determine the energy transfer rate.

Following the analysis of the steady-state radiation case a discussion of transient radiation energy-transfer is undertaken. The results are less general than for the steady-state case, because of the more complicated nature of the equations.

MATHEMATICAL FORMULATION

For the problems considered it is assumed that

* A recent publication by Lick [14] examines transient radiation transport in a semi-infinite medium for small temperature gradients.

the medium is in local thermodynamic equilibrium and therefore the local temperature along with the physical properties of the medium completely determine the energy transfer. Since numerous derivations of the equations describing one dimensional radiative energy transfer in a bounded medium are available [6–11], development is omitted and the results are taken directly from these articles. The equation of transfer for the radiation intensity between the plates is:

$$\mu \frac{\mathrm{d}I_\nu}{\mathrm{d}y} = a_\nu[B_\nu(T) - I_\nu]. \quad (1)$$

where $\mu = \cos\theta$ as shown in Fig. 1. By introducing the variable

$$\tau_\nu = \int_0^y a_\nu \,\mathrm{d}y$$

this equation is transformed into:

$$\mu \frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = B_\nu(T) - I_\nu. \quad (2)$$

For simplicity the frequency subscript will be omitted on τ_ν with the understanding that τ is always frequency dependent.

If I_ν satisfies the boundary conditions $I_\nu = I_\nu(0)$ at $y = 0$ and $I_\nu = I_\nu(l)$ at $y = l$ then the radiative flux between $y = 0$ and $y = l$ can be obtained by solving equation (2). The steps involved in this are given in [11] and the result is found to be:

$$\frac{q_r}{\pi} = 2\int_0^\infty [R_{1,\nu}E_3(\tau) - R_{2,\nu}E_3(\tau_0 - \tau) + \int_0^\tau B_\nu E_2(\tau - t)\,\mathrm{d}t - \int_\tau^{\tau_0} B_\nu E_2(t - \tau)\,\mathrm{d}t]\,\mathrm{d}\nu. \quad (3)$$

In this expression $2\pi R_{1,\nu}$ and $2\pi R_{2,\nu}$ are the radiosities of the surface and are defined by the equations:

$$2\pi R_{1,\nu} = \{\varepsilon_1 B_\nu(T_1) + 2(1 - \varepsilon_1)[R_{2,\nu}E_3(\tau_0) + \int_0^{\tau_0} B_\nu E_2(t)\,\mathrm{d}t]\}\,2\pi \quad (4)$$

$$2\pi R_{2,\nu} = \{\varepsilon_2 B_\nu(T_2) + 2(1 - \varepsilon_2)[R_{1,\nu}E_3(\tau_0) + \int_0^{\tau_0} B_\nu E_2(\tau_0 - t)\,\mathrm{d}t]\}\,2\pi. \quad (5)$$

In the radiosities we have assumed that the emissivity is frequency independent and that the surfaces reflect diffusely. The function $E_n(t)$ appearing in equations (3), (4) and (5) is defined as:

$$E_n(t) = \int_0^1 \mu^{n-2} \exp(-t/\mu)\,\mathrm{d}\mu. \quad (6)$$

The equation describing the temperature distribution between the plates, provided there is no flow, is the classical expression:

$$\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) = \rho c_v \frac{\partial T}{\partial t} + \frac{\partial q_r}{\partial y}, \quad (7)$$

where $-\,\partial q_r/\partial y$ is the energy source term produced in the medium by the radiation field. The heat flux to the surface $y = 0$ is given by the expression:

$$q = -\left(k\frac{\partial T}{\partial y}\right)_{y=0} + 2\pi\int_0^\infty \left[\frac{R_{1,\nu}}{2} - R_{2,\nu}E_3(\tau_0) - \int_0^{\tau_0} B_\nu E_2(t)\,\mathrm{d}t\right]\mathrm{d}\nu. \quad (8)$$

Our objective now will be to solve equation (7) analytically when coupled with equations (4), (5) and (3) for various conditions on the parameters of the problem. Again we note that it is assumed that the absorption coefficient a_ν can be described by an equation of the form $a_\nu = \beta(T)\,k_\nu$. In this expression the function β accounts for the temperature dependence and k_ν for the spectral variations. The opacity τ_0 will be required to satisfy the condition $\tau_0 < 1$ at all frequencies.

SOLUTIONS FOR VARIOUS CONDITIONS

1. *Pure radiation*

In the event that conduction, convection and transient effects are negligible, equation (7) describing the energy-transfer in the gas becomes $\partial q_r/\partial y = 0$ which, by employing equation (3), may be written as

$$\int_0^\infty [-R_{1,\nu}E_2(\tau) - R_{2,\nu}E_2(\tau_0 - \tau) + 2B_\nu(T) - \int_0^\tau B_\nu E_1(\tau - t)\,\mathrm{d}t - \int_\tau^{\tau_0} B_\nu E_1(t - \tau)]\,k_\nu\,\mathrm{d}\nu = 0. \quad (9)$$

If we now expand equations (4), (5) and (9) for small τ_0 we find

$$\frac{2}{e_2}\int_0^\infty B_\nu(T)\,k_\nu\,d\nu = \frac{(2-\varepsilon_2)\,\varepsilon_1}{e_2 g}\int_0^\infty B_\nu(T_1)\,k_\nu\,d\nu + \frac{(2-\varepsilon_1)\,\varepsilon_2}{e_2 g}\int_0^\infty B_\nu(T_2)\,k_\nu\,d\nu + 0(\tau_0 \ln \tau_0) \quad (10)$$

where $g = [1 - (1-\varepsilon_1)(1-\varepsilon_2)]$.

Equation (10) is an implicit equation for the temperature T provided one knows the spectral distribution function, k_ν. This can be solved in a straightforward manner, since all the terms on the right-hand side are known and one only has to choose a value of T which when employed in evaluating $B_\nu(T)$ will yield an equality.

Assuming now that the solution has been determined it is of interest to write equation (10) in terms of Planck means for comparison with a gray gas analysis. For this purpose we will denote the temperature T which satisfies equation (10) as T_0. Recalling the definition of a Planck mean to be:

$$k_{p,i} = \pi \int_0^\infty B_\nu(T_i)\,k_\nu\,d\nu/\sigma T_i^4$$

equation (10) can be written as:

$$\frac{\sigma T_0^4}{e_2} = \frac{k_{p,1}(2-\varepsilon_2)\,\varepsilon_1 \sigma T_1^4}{2k_{p,0}\,g e_2} + \frac{k_{p,2}(2-\varepsilon_1)\,\varepsilon_2 \sigma T_2^4}{2k_{p,0}\,g e_2} + 0(\tau_0 \ln \tau_0). \quad (11)$$

This result shows that three different Planck means are required to determine the temperature in a non-gray medium. A gray gas-analysis would require that $k_{p,1} = k_{p,2} = k_{p,0}$ which is true only if $T_1 = T_2$ or k_ν is frequency independent. Note also that the temperature dependence, $\beta(T)$ of the absorption coefficient did not enter into the evaluation of the temperature.

It is of interest to take a simple example and determine the magnitude of error which would exist between the temperature predicted by equation (11) and that by a gray gas analysis. Let us assume that k_ν has the simple spectral form:

$$k_\nu = 1 \text{ for } 0 \leqslant \nu \leqslant \nu_0;$$
$$k_\nu = 0 \text{ for } \nu > \nu_0.$$

If we also assume that $h\nu_0/\bar{k}T$ is always small we can show that:

$$k_{p,i} = \pi \int_0^\infty B_\nu k_\nu\,d\nu \Big/ \sigma T_i^4 \approx \frac{5}{\pi^4}\left(\frac{h\nu_0}{\bar{k}T_i}\right)^3.$$

Inserting this result into equation (11) and setting $\varepsilon_1 = \varepsilon_2 = 1$ yields:

$$T_0 = \frac{T_1 + T_2}{2},$$

while a gray gas analysis predicts that [3]:

$$T_0 = \left[\frac{T_1^4 + T_2^4}{2}\right]^{\frac{1}{4}}.$$

Figure 2 compares these two results for various values of the ratio (T_1/T_2). We see that indeed a

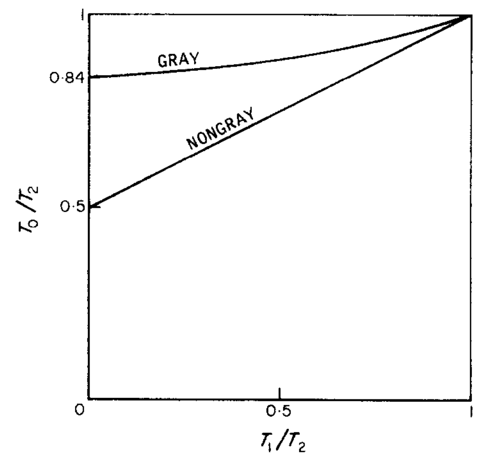

FIG. 2. Comparison of temperatures predicted by gray and non-gray analysis.

significant difference can exist between the gray gas analysis and the spectral analysis.

We can now extend the analysis to the next higher order term in τ_0 by approximating the temperature dependence of the opacity with the first order solution, T_0. We then find that:

$$\tau = \int_0^y \beta(T_0)\, k_\nu\, dy = \beta(T_0)\, k_\nu y.$$

This step provides a convenient simplification which will permit us to continue the analysis without extreme difficulty.

In completing the analysis the expressions become extremely cumbersome if we maintain emissivities different from one. Although the analysis has been carried out without this restriction, results are given here only for $\varepsilon_1 = \varepsilon_2 = 1$. The radiosities with the emissivities set equal to one become:

$$R_{1,\nu} = B_\nu(T_1);$$

$$R_{2,\nu} = B_\nu(T_2).$$

Using these radiosities and following a straightforward successive approximation expansion we arrive at an expression for T which, when written in terms of mean coefficients has the form:

$$\frac{\pi 2}{e_2}\int_0^\infty B_\nu(T)\, k_\nu\, d\nu = \frac{2\sigma T^4 k_p}{e_2} = \frac{1}{e_2}\{\sigma T_1^4 k_{p,1} + \sigma T_2^4 k_{p,2} + \beta(T_0)\, y[\gamma - 1 + \ln(\beta(T_0)\, y)].$$
$$\times [\sigma T_1^4 k_{m,1}^2 - \sigma T_0^4 k_{m,0}^2]$$
$$+ \beta(T_0)\, y[\sigma T_1^4 k_{q,1}^2 \ln k_{q,1} - \sigma T_0^4 k_{q,0}^2 \ln k_{q,0}]$$
$$- \beta(T_0)(y - l)[\gamma - 1 + \ln \beta(T_0)(l - y)][\sigma T_2^4 k_{m,2}^2 - \sigma T_0^4 k_{m,0}^2]$$
$$+ \beta(T_0)(l - y)[\sigma T_2^4 k_{q,2}^2 \ln k_{q,2} - \sigma T_0^4 k_{q,0}^2 \ln k_{q,0}]\}$$
$$+ O(\tau_0^2 \ln \tau_0). \qquad (12)$$

The γ which appears in this expression arises from the asymptotic expansions of the functions defined by equation (6).

In equation (12) new mean absorption coefficients appear which to the author's knowledge have not been computed. These are defined by the equations:

$$k_{m,i}^2 = \pi \int_0^\infty B_\nu(T_i)\, k_\nu^2\, d\nu/\sigma T_i^4; \qquad (13)$$

$$k_{q,i}^2 \ln k_{q,i} = \pi \int_0^\infty B_\nu(T_i)\, k_\nu^2 \ln k_\nu\, d\nu/\sigma T_i^4. \qquad (14)$$

Equation (12) shows that in order to describe the temperature of a non-gray gas to terms of order $(\tau_0^2 \ln \tau_0)$ nine different mean absorption coefficients are required.

Next let us examine the heat flux to the surface $y = 0$ to determine how sensitive this result will be to spectral absorption. Since conduction is assumed absent the heat flux is given by the expression:

$$q_r = 2\pi \int_0^\infty \left[\frac{R_{1,\nu}}{2} - R_{2,\nu} E_3(\tau_0) - \int_0^{\tau_0} B_\nu E_2(t)\, dt\right] d\nu.$$

Due to the fact that the heat flux is generally the most useful quantity the results presented will include emissivities. Upon introducing the expressions for $R_{1,\nu}$ and $R_{2,\nu}$ and carrying out the expansion the resulting expression for q_r becomes:

$$\frac{q_r}{e_2} = \frac{\varepsilon_1\varepsilon_2(\sigma T_1^4 - \sigma T_2^4)}{e_2 g} + \left\{\frac{[(2+f)\,\varepsilon_1]\,\varepsilon_2 k_{p,2}}{g}\sigma T_2^4 - \frac{[(4+f)\,\varepsilon_2 - 4]\,\varepsilon_1 k_{p,1}}{g}\sigma T_1^4\right.$$
$$\left. - 2(\varepsilon_1 - \varepsilon_2 + 1)\,\sigma T_0^4 k_{p,0}\right\}\frac{\beta(T_0)}{e_2} l + O(\tau_0^2 \ln \tau_0). \qquad (15)$$

where: $f = 4(1 - \varepsilon_1)(1 - \varepsilon_2)/g$.

This result shows the interesting fact that a gray gas assumption will predict the heat transfer in the optically thin limit [9], but fails for higher order corrections. We find that the three Planck means required to predict T_0 to of order $\tau_0 \ln \tau_0$ are the only ones required to obtain the heat flux to of order $\tau_0^2 \ln \tau_0$.

In the limit of $\varepsilon_1 = \varepsilon_2 = 1$ we find, to of order $\tau_0^2 \ln \tau_0$, that:

$$q_r = (\sigma T_1^4 - \sigma T_2^4) + (\sigma T_2^4 k_{p,2} - \sigma T_1^4 k_{p,1})\,\beta(T_0)\,l.$$

This result provides us with a convenient result for comparing with a gray gas analysis. If we use the same spectral distribution for k_ν that was used to compare T_0 we find that our heat flux expression reduces to:

$$q_r = (\sigma T_1^4 - \sigma T_2^4) + (\sigma T_2 - \sigma T_1)\left(\frac{h\nu_0}{k}\right)^3 \frac{5\beta(T_0)}{\pi^4}\,l.$$

Using a Planck mean based on the first order temperature

$$T_{0,g} = \left(\frac{T_1^4 + T_2^4}{2}\right)^{\frac{1}{4}}$$

of gray medium analysis we can write the gray gas heat flux given by Cess [3] in the form:

$$q_{r,g} = (\sigma T_1^4 - \sigma T_2^4) + (\sigma T_2^4 - \sigma T_1^4)\left(\frac{h\nu_0}{kT_{0,g}}\right)^3 \frac{5\beta(T_{0,g})}{\pi^4}\,l.$$

For a simple comparison let us take $\beta(T) =$ constant so that we can write:

$$F = (1 - T_1/T_2), \text{ and}$$

$$F_g = 2^{\frac{3}{4}} \frac{[1 - (T_1/T_2)^4]}{[1 + (T_1/T_2)^4]^{\frac{3}{4}}},$$

$$\text{where } F = \pi^4[q_r - (\sigma T_1^4 - \sigma T_2^4)]/\sigma\left(\frac{h\nu_0}{k}\right)^3 5\beta l T_2.$$

These two expressions are compared on Fig. 3. We see that the gray gas analysis can under predict the heat transfer considerably. If we included a variable $\beta(T)$ then this comparison may become better or worse depending on the functional form of the temperature dependence.

2. *Radiation with unsteady wall temperatures*

Consider next the situation of a time varying temperature at the surface $y = 0$. In this case the equation describing the temperature field in the medium is:

$$\rho c_v \frac{\partial T}{\partial t} = 2\beta \int_0^\infty \pi\left[\frac{R_{1,\nu}}{2} + \frac{R_{2,\nu}}{2} - 2B_\nu(T)\right] k_\nu \, d\nu + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + 0\left(\frac{e_2\tau_0^2}{l}\ln \tau_0\right). \quad (16)$$

The initial and boundary conditions which will be imposed are:

$T = T_2$ for $t = 0$ and all y; $T = T_1$ at $y = 0$, $T = T_2$ at $y = l$ for $t > 0$.

This equation has two characteristic response times, one for the conduction and the other for the radiation transport. If we examine these we find the characteristic time for conduction near the wall $y = 0$ will be given by:

$$t_c = \rho c_v y^2 T_2/kT_2,$$

and near the wall $y = l$,

$$t_c = \rho c_v (l - y)^2\, T_2/kT_2.$$

The radiation transport time is given by:

$$t_r = \rho c_v T_2/\beta k_\nu e_2.$$

If we require now that t_c be greater than t_r we can neglect conduction effects in the energy equation provided of course the actual time t remains less than t_c. Assuming now that $t_c > t_r$ we can write:

$$\frac{\partial T}{\partial t} = \frac{2\beta}{\rho c_v}\int_0^\infty \pi\left[\frac{R_{1,\nu}}{2} + \frac{R_{2,\nu}}{2} - 2B_\nu(T)\right] k_\nu\, d\nu + 0\left(\frac{T_2}{t_r}\tau_0 \ln \tau_0\right). \tag{17}$$

A first integral of this equation is:

$$T = T_2 + \int_0^t \frac{2\beta}{\rho c_v}\int_0^\infty \pi\left[\frac{R_{1,\nu}}{2} + \frac{R_{2,\nu}}{2} - 2B_\nu(T)\right] k_\nu\, d\nu\, dt + 0\left(\frac{T_2 t}{t_r}\tau_0 \ln \tau_0\right).$$

Now if we employ successive approximations we obtain the result that:

$$T = T_2 + \frac{\pi 2\beta(T_2)}{(\rho c_v)_{T_2}}\left[\frac{(2-\varepsilon_2)\,\varepsilon_1 \sigma T_1^4 k_{p,1}}{2g\pi} + \frac{(2-\varepsilon_1)\,\varepsilon_2 \sigma T_2^4 k_{p,2}}{2g\pi} - \frac{2\sigma T_2^4 k_{p,2}}{\pi}\right] t$$
$$+ 0\left(T_2 \frac{t}{t_r}\tau_0 \ln \tau_0\right) + 0\left[T_2\left(\frac{t}{t_r}\right)^2\right]. \tag{18}$$

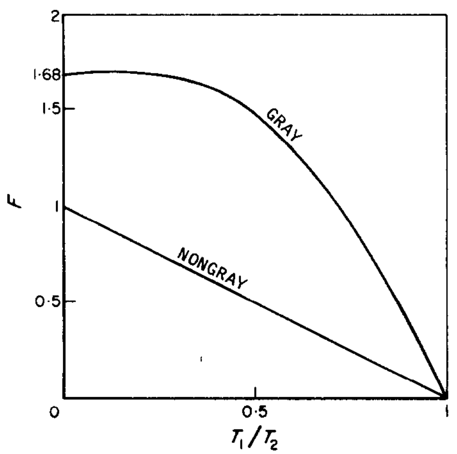

FIG. 3. Comparison of heat flux terms for gray and non-gray analysis.

This expression gives the temperature distribution in the medium for small times provided $t_c > t_r > t$. Since we assumed the gas to be optically thin to a first approximation, and neglected conduction the temperature of the gas is predicted to vary uniformly between the plates with time and to have a discontinuity at $y = 0$ and $y = l$. The predicted temperature distribution will be accurate except close to the walls where t_c begins to approach t_r or equivalently when the distance from the wall becomes less than $y = [kT_2/(\beta k_\nu)e_2]^{\frac{1}{2}}$. When this occurs the conduction terms will begin to dominate the energy transport with the result that the temperature discontinuities predicted by equation (18) will be smoothed into a continuous temperature distribution. This same phenomenon occurs in steady radiation transport problems when conduction is small compared to radiation as pointed out by Cess [3] and Lick [13]. Before heat transfer to the walls can be calculated the conduction term must be included since the heat flux is a function of both position and time near the walls. In this paper we will not attempt to include conduction effects.

SUMMARY

The principal results of this paper are given in equations (11), (12), (15) and (18). Equation (11) shows the important result that for an optically thin gas ($\tau_0 \ll 1$) the temperature for a radiation dominated energy transfer is characterized by three different Planck mean coefficients. This result demonstrates the limitations of a gray gas analysis since such an analysis predicts that the temperature is independent of the absorption coefficient. Equation (12) extends the solution for the temperature, given to first order by equation (11), to a higher order in τ_0. It is found that a new group of mean absorption coefficients must be introduced to describe the temperature distribution. These new means point strongly to the inappropriateness of using a gray gas assumption to compute the temperature distribution in radiation dominated flow. They also point out that one cannot simply substitute Planck means in a gray gas analysis and expect to obtain good accuracy.

Equation (15) is an expression for the heat flux in a radiation dominated energy transfer problem. It shows that for $\tau_0 \to 0$ a gray gas assumption is appropriate for calculating heat transfer. It also shows however that if terms of order τ_0 are included then three different Planck means are required to determine the heat transfer and hence the gray gas assumption becomes inappropriate.

Equation (18) gives the transient temperature distribution between two plates for small times subject to the initial and boundary conditions of equation (13). This is for an optically thin gas in which radiation dominates the energy transfer. It is important to note that this expression fails very near the walls because conduction becomes dominant.

In applying the results of this paper it is important for the user to stay within the bounds of the assumptions. The four principal restrictions of the analysis are (1) $\tau_\nu < 1$, (2) the absorption coefficient can be expressed in the form $a_\nu = \beta(T)k_\nu$, (3) emissivities of the surfaces are frequency independent, (4) radiation is the dominant mechanism of energy transfer. All of these assumptions can be realized physically, but from a practical point of view, the principal limitation on the results is the requirement that the optical depth be less than one. The other approximations are adequate, however, for a variety of problems.

In conclusion it should be noted that if k_ν is a complex function of frequency it may be necessary to employ a electronic computer to evaluate the integrals involving this function.

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Résumé—On a étudié théoriquement le transport d'énergie d'un milieu non-gris limité par deux surfaces plates. Le modèle théorique emploie un coefficient d'absorption dépendant de la longueur d'onde et de la température au lieu de l'hypothèse classique du gaz gris, L'étude tient compte de l'effet des émissivités superficielles et des conditions transitoires. L'analyse a été conduite sous l'hypothèse restrictive que la profondeur optique τ_ν, est inférieure à l'unité. Des expressions pour la distribution de température et le flux de chaleur sont présentées. Les résultats montrent les insuffisances du modèle du gaz gris.

Zusammenfassung—Es wird eine theoretische Untersuchung durchgeführt über den Energietransport in einem nichtgrauen Medium, das von zwei ebenen Oberflächen begrenzt wird. Das theoretische Modell weist einen spektralen und temperaturabhängigen Absorptionskoeffizienten auf, anstelle der klassischen Annahme von grauem Gas. Die Arbeit umfasst die Einflüsse von Oberflächenemission und Übergangsbedingungen. Die Analyse wird unter der Einschränkung durchgeführt, dass die optische Tiefe τ_ν kleiner also eins ist. Sowohl für die Temperaturverteilung als auch für den Wärmefluss sind Formeln angegeben. Die Ergebnisse zeigen die Unzulänglichkeit des grauen Gasmodells.

Аннотация—Проведено теоретическое исследование переноса энергии в несерой среде, ограниченной двумя плоскими поверхностями. В теоретической модели используется коэффициент абсорбции, зависящий от спектра и температуры, вместо классического допущения о сером газе. Учитывались влияния излучательной способности поверхности и переходные условия. Анализ проводился при допущении, что оптическая глубина τ_ν меньше единицы. Даются выражения для температурного распределения и теплового потока. Полученные результаты демонстрируют неудовлетворительность модели серого газа.